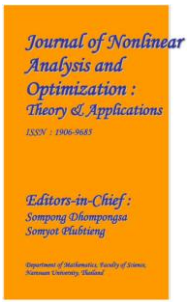
Journal of **N**onlinear **A**nalysis and **O**ptimization
Vol. 15, Issue. 2, No.3 : 2024
ISSN : **1906-9685**



A NEW APPROACH TO FIND FUZZY CRITICAL PATH IN INTUITIONISTIC HEPTAGONAL FUZZY NUMBER

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ABSTRACT:
In this paper, an alternate approach were introduced to evaluate possible critical path using Intuitionistic Heptagonal Fuzzy Number(IHFN) and an innovative ranking technique is applied to defuzzify the IHFN .Here an alternate algorithm was framed to find the longest path. It was observed that the proposed methods were very effective in calculating the critical path and an illustrative example were given to demonstrate the above procedure.
KEYWORDS: IHFN, Ranking of Heptagonal Fuzzy Number (HFN), Ranking of IHFN.

INTRODUCTION:

Nowadays most of the researchers are interested in solving mathematical problems using networks. In some situations it is found that it is not possible to identify all the numbers to be as fuzzy numbers in such cases Intuitionistic Fuzzy Number (IFN) arise where Truth and False membership function occurs. All the real time situations are modelled as Fuzzy Set Theory which was emerged by Zadeh [13]. According to PERT computations, the directed network indicates the precedence relationship array for various activities.

In Networks determining possible values of each path by means of the formula were done by many researchers. S.Priyadarshini and G. Deepa[4] introduced a new method by using the maximum technique for edge distance to determine the path. N.Ravishankar [6] proposed a method to evaluate the characteristics to develop the longest path. In this way, V.Siresha and N. Ravishankar [7] determined multiple possible paths to detect the fuzzy network. According to this C.Rajendran [5] come out of their ideas in evaluating critical path in hexagonal numbers.. L. Sujatha [11] evaluated the IFN to calculate the shortest path In this paper an innovative technique was used to detect the critical path and a new ranking techniques was applied according to the criteria

1. BASIC DEFINITION:

1.1 FUZZY SET:

A fuzzy set A⊆ X which assigns the value in the interval [0,1] for all x∈X, where
$\mu_{\check{A}}$is called the membership function of X, where$\check{A}$ = $\{(x, \mu_{\check{A}}(x));\ x \in X\}$ defined by $\mu_{\check{A}}(x)$ for each x∈X, is called a fuzzy set.

1.2 FUZZY NUMBER:

A membership function $\mu_{\check{A}}(x)$: R→ [0,1] is called the membership function if it satisfies the following characteristics.

(i) $\mu_{\check{A}}(x)$ is piecewise continuous
(ii) $\mu_{\check{A}}(x)$ is a convex fuzzy subset.
(iii) $\mu_{\check{A}}(x)$ is normal (i,e) there exists $x_0 \in R$ such that $\mu_{\check{A}}(x_0)$ =1

2. INTUITIONISTIC FUZZY NUMBER:

The set of real numbers defined on the fuzzy set is called intuitionistic fuzzy number if $\tilde{A}$ = $\{(x, \mu_{\tilde{A}}(x), \gamma_{\tilde{A}}(x)) : x \in R\}$

(i)There exist a real number $x_0 \in R$ such that $\mu_{\tilde{A}}(x_0) = 1$
and $\gamma_{\tilde{A}}(x_0) = 0$
(ii) $\mu_{\tilde{A}}$of $\widetilde{A}$ is fuzzy convex and upper semi-continuous.
(iii) $\gamma_{\tilde{A}}$ of A is fuzzy concave and lower semi-continuous.

2.1 HEPTAGONAL FUZZY NUMBER:

A fuzzy number $\check{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7)$ is a heptagonal fuzzy number where $a_1, a_2, a_3, a_4, a_5, a_6, a_7$ are real numbers and $a_1 < a_2 < a_3 < a_5 < a_6 < a_7$ and its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{1}{3}\left(\frac{x-a_1}{a_2-a_1}\right) for\ a_1 \le x \le a_2 \\ \frac{1}{3} + \frac{1}{3}\left(\frac{x-a_2}{a_3-a_2}\right) for\ a_2 \le x \le a_3 \\ \frac{2}{3} + \frac{1}{3}\left(\frac{x-a_3}{a_4-a_3}\right) for\ a_3 \le x \le a_4 \\ 1 - \frac{1}{3}\left(\frac{x-a_4}{a_5-a_4}\right) for\ a_4 \le x \le a_5 \\ \frac{2}{3} - \frac{1}{3}\left(\frac{x-a_5}{a_6-a_5}\right) for\ a_5 \le x \le a_6 \\ \frac{1}{3}\left(\frac{a_7-x}{a_7-a_6}\right) for\ a_6 \le x \le a_7 \\ 0\ \ for\ x < a_1\ and\ x > a_7 \end{cases}$$

2.2 RANKING FUNCTION FOR MEMBERSHIP FUNCTION OF A HEPTAGONAL FUZZY NUMBER:

Ranking function is the most important concept to compare the fuzzy numbers to our fuzzy approach R: F(R) → R where F(R) is a set of fuzzy numbers defined on a set of real numbers which maps fuzzy number into real space.

Here

[P(α),P'(α)] = [3α$(a_2 - a_1)$+ a_1, -3α$(a_7 - a_6)$ + a_7] for α ∈ (0, $\frac{1}{3}$)

[Q(α),Q'(α)] = [3(α $-\frac{1}{3}$) $(a_3 - a_2)$ +a_2 , -3(α $-\frac{2}{3}$) $(a_6 - a_5)$ +a_5]

for α ∈ ($\frac{1}{3}$,$\frac{2}{3}$)

[R(α),R'(α)] = [3(α $-\frac{2}{3}$) $(a_4 - a_3)$ +a_3 , -3(α $-$ 1) $(a_5 - a_4)$ +a_4]

for α ∈ ($\frac{2}{3}$, 1)

$= \frac{a_1 + 6a_2 + 12a_3 + 16a_4 + 12a_5 + 6a_6 + a_7}{54}$

3. NON –MEMBERSHIP FUNCTION FOR A HEPTAGONAL FUZZY NUMBER:

A heptagonal intuitionistic fuzzy number $\breve{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7)$ is a heptagonal intuitionisticfuzzy number where $a_1, a_2, a_3, a_4, a_5, a_6, a_7$ are real numbers and $a_1 < a_2 < a_3 < a_5 < a_6 < a_7$ and its non- membership function is given by

$$\gamma_{\tilde{A}}(x) = \begin{cases} 1 - \frac{1}{3}\left(\frac{x-a_1}{a_2-a_1}\right) for\ a_1 \le x \le a_2 \\ \frac{2}{3} - \frac{1}{3}\left(\frac{x-a_2}{a_3-a_2}\right) for\ a_2 \le x \le a_3 \\ \frac{1}{3}\left(\frac{a_3-x}{a_4-a_3}\right) for\ a_3 \le x \le a_4 \\ 0\ for\ x = a_4 \\ \frac{1}{3}\left(\frac{x-a_4}{a_5-a_4}\right) for\ a_4 \le x \le a_5 \\ \frac{1}{3} + \frac{1}{3}\left(\frac{x-a_5}{a_6-a_5}\right) for\ a_5 \le x \le a_6 \\ \frac{2}{3} + \frac{1}{3}\left(\frac{x-a_6}{a_7-a_6}\right) for\ a_6 \le x \le a_7 \\ 1\ \ for\ x > a_7 \end{cases}$$

3.1 RANKING FUNCTION FOR NON-MEMBERSHIP FUNCTION OF A HEPTAGONAL FUZZY NUMBER:

P(α),P'(α)] = [-3(α-1)$(a_2 - a_1)$ + a_1, 3(α-$\frac{2}{3}$)$(a_7 - a_6)$ + a_6]

for α ∈ (0, $\frac{1}{3}$)

[Q(α),Q'(α)] = [-3(α − $\frac{2}{3}$) $(a_3 - a_2)$ +a_2 , 3(α − 1) $(a_6 - a_5)$ +a_5]

for α ∈ ($\frac{1}{3}$,$\frac{2}{3}$)

[R(α),R'(α)] = [-3(α$(a_4 - a_3)$+ a_4, 3(α$(a_5 - a_4)$ +a_4]

for α ∈ ($\frac{2}{3}$, 1)

= $\frac{-4a_1+12a_2+42a_3-46a_4+42a_5+12a_6-4a_7}{54}$

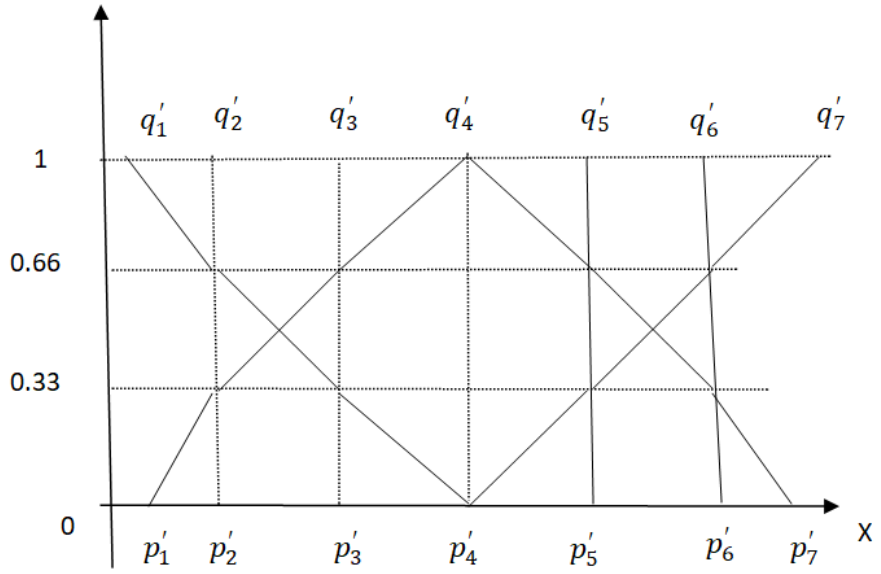

Fig1: Intuitionistic heptagonal fuzzy number

4. ARITHMETIC OPERATIONS ON HEPTAGONAL FUZZY NUMBER:

Let $\widetilde{A_1}$= $(a_1, b_1, c_1, d_1, e_1, f_1, g_1)$ and $\widetilde{A_2}$ = $(a_2, b_2, c_2, d_2, e_2, f_2, g_2)$ be two heptagonal fuzzy numbers, then there operations are

4.1 ADDITION:

$\check{A}_1 + \check{A}_2$ = $(a_1 + a_2 , b_1 + b_2 , c_1 + c_2, d_1 + d_2 , e_1 + e_2, f_1 + f_2, g_1 + g_2)$

4.2 SUBTRACTION:

$\check{P} - \acute{Q}$= ($p_1' - q_1'$, $p_2' - q_2'$, $p_3' - q_3'$, $p_4' - q_4'$, $p_5' - q_5'$, $p_6' - q_6'$, $p_7' - q_7'$)

5. PROPOSED ALGORITHM TO FIND FUZZY CRITICAL PATH:

Step 1: Find all the possible critical paths by using the arithmetic operation of addition in heptagonal fuzzy number. Calculate the length of the path.

Step 2: Then the new ranking technique is applied and calculated for each possible paths.

For membership function the ranking value is

$$\frac{a_1 + 6a_2 + 12a_3 + 16a_4 + 12a_5 + 6a_6 + a_7}{54}$$

And for non-membership function the ranking value is

$$\frac{-4a_1 + 12a_2 + 42a_3 - 46a_4 + 42a_5 + 12a_6 - 4a_7}{54}$$

Step 3: Then the ranking is calculated by

max($\frac{a_1+6a_2+12a_3+16a_4+12a_5+6a_6+a_7}{54}$)

and max ($\frac{-4a_1+12a_2+42a_3-46a_4+42a_5+12a_6-4a_7}{54}$)

by using this criteria based on the the rank the possible path is confirmed and this path is considered as the critical path.

5.1 NUMERICAL EXAMPLE:

In a shoe manufacturing company there are different nodes in the manufacturing section. In this way the delay of any activity on the path will be the highest risk of the project cannot be finished on time. So the network representation is used to display the relationship between the project activities and to help managers to address all the way of questioning.

Node 1: Creative endeavour design
Node 2: Suitable pattern making is selected
Node 3: Cutting and stitching technique
Node 4: Assembling in format order
Node 5: Finishing adds aesthetic value to the final product
Node 6: Final inspection to check the defects or issues.

Table 1: The table showing the activity and path duration of heptagonal Fuzzy number

Activity	Duration
1 – 2	(3,4,6,7,7.2,8,9) (1,4,6,7,8,9,10)
1 – 3	(2,3,3.3,4,5,6,8) (1,3,3.3,5,7,8,9)
1 – 5	(3,4,5,6,7,8,9) (2,4,5,6,8,9,10)
2 – 3	(2,3,3.6,4,5,6,8) (1,3,3.6,5,6,7,9)
2 – 4	(2,4,5,6,7,8,10) (1,4,5,7,,9,10,11)
3 – 4	(2,3,4,4.4,5,7,8) (1,3,4,6,6.8,7,9)
4 – 6	(2,2.5,3,4,5,7,9) (1,2.5,3,4,5,6,10)
5 – 6	(4,5,5.7,6,7,8,9) (3,5,5.7,7,8.4,9,10)

The network diagram for the above data

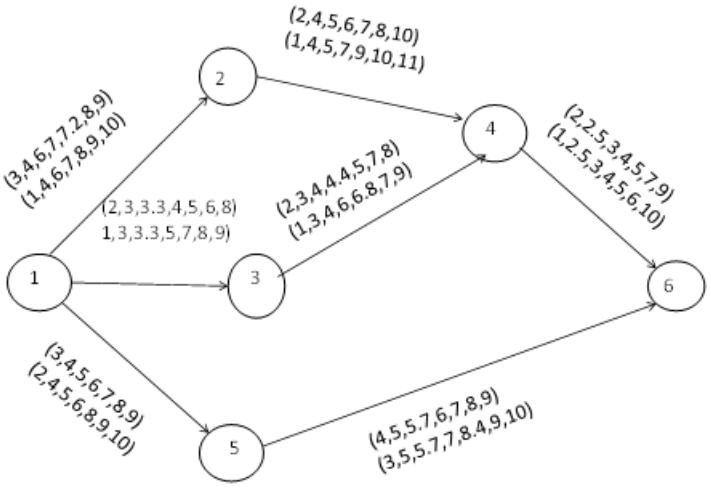

Fig 2: Network diagram of heptagonal intuitionistic fuzzy number

The possible critical paths are

1→ 2 → 4 → 6
1→ 2 → 3 → 4 → 6

1→ 3 → 4 → 6
1→ 5 → 6

5.2 CALCULATION FOR THE NUMERICAL EXAMPLE:

Step 1:

The length of the path is calculated using arithmetic operation
Addition operation on heptagonal fuzzy number
$\breve{P}_1 + \breve{Q}_1 = (p_1' + q_1',\ p_2' + q_2',\ p_3' + q_3', p_4' + q_4',\ p_5' + q_5',\ p_6' + q_6',\ p_7' + q_7')$
For the path 1→ 2 → 4 → 6 we have to calculate addition operation for
(3,4,6,7,7.2,8,9) (1,4,6,7,8,9,10) + (2,4,5,6,7,8,10) (1,4,5,7,9,10,11) + (2,2.5,3,4,5,7,9) (1,2.5,3,4,5,6,10) = (7,10.5,14,17, 19.2, 23, 28) (3, 9.5,12,17, 22, 25, 31)

Step 2:

Now the ranking procedure for intuitionistic heptagonal fuzzy number is calculated for membership function as

$R(p_1', p_2', p_3', p_4', p_5', p_6', p_7') = \max\ \left(\frac{p_1'+6p_2'+12p_3'+16p_4'+12p_5'+6p_6'+p_7'}{54}\right)$

and for non-membership function as

$R(q_1', q_2', q_3', q_4', q_5', q_6', q_7') = \max\ \left(\frac{-4q_1'+12q_2'+42q_3'-46q_4'+42q_5'+12q_6'-4q_7'}{54}\right)$

Now the ranking procedure for the membership function is calculated for (7, 10.5,14,17, 19.2, 23, 28) (3, 9.5,12,17,22,25,31) as

$R(p_1', p_2', p_3', p_4', p_5', p_6', p_7') = \max\ \left(\frac{p_1'+6p_2'+12p_3'+16p_4'+12p_5'+6p_6'+p_7'}{54}\right)$

$= \frac{7+63+168+272+230.4+138+28}{54}$

$= \frac{906.4}{54} = 16.785$

Then the ranking procedure for non-membership function is given by

$R(q_1', q_2', q_3', q_4', q_5', q_6', q_7') = \max\ \left(\frac{-4q_1'+12q_2'+42q_3'-46q_4'+42q_5'+12q_6'-4q_7'}{54}\right)$

$= \frac{-12+114+504-782+924+300-148}{54}$

$= \frac{900}{54} = 16.666$

Step 3:

In a similar way we calculate the ranking for the remaining possible paths

Table2: Table showing the ∝ - cut ranking values

Possible paths	Length of the path	∝ - cut ranking technique	Ranking
1→ 2 → 4 → 6	(7,10.5,14,17, 19.2, 23,28) (3, 9.5, 12, 17, 22, 25, 31)	(16.785, 16.666)	2
1→ 2 → 3 → 4 → 6	(9,12.5,16.6,19.4,22.2,28,34)(4, 12.5, 16.6, 22, 25.8,29,38)	(19.666, 20.348)	1
1→ 3 → 4 → 6	(6, 8.5, 10.3,12.4,15, 20, 25) (3,8.5,10.3,15,18.8, 21, 28)	(13.037, 14.114)	3
1→ 5 → 6	(7, 9,10.7, 12, 14, 16,18) (5, 9, 10.7,13,16.4,18, 20)	(12.285, 14.15)	4

Among the paths [max($\mu_{\tilde{A}}(x)$, max($\gamma_{\tilde{A}}(x)$] is used in α – cut ranking technique then that path is considered as critical path.from this table we concluded that the critical path is 1→ 2 → 3 → 4 → 6

6. CONCLUSION:

In this paper the critical path have been computed using the new ranking technique for membership and non-membership function of Heptagonal Fuzzy Number.Although many researchers have focused on critical path the proposed new algorithmic procedure gives a detailed analysis of Intuitionistic Heptagonal Fuzzy number. Hence the procedures developed in this paper were an alternate algorithm to calculate the critical path and it was explained with a real time example.

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